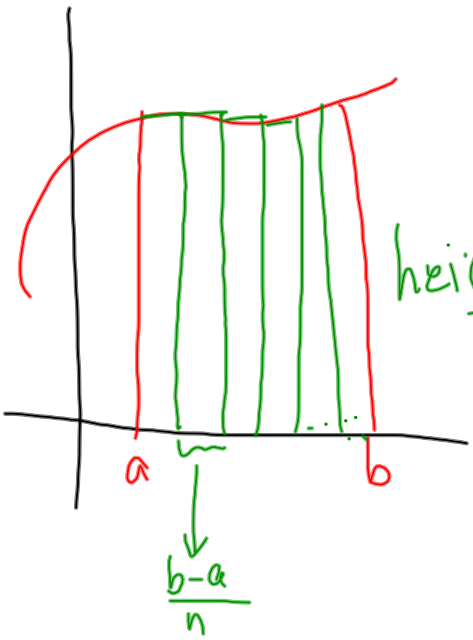
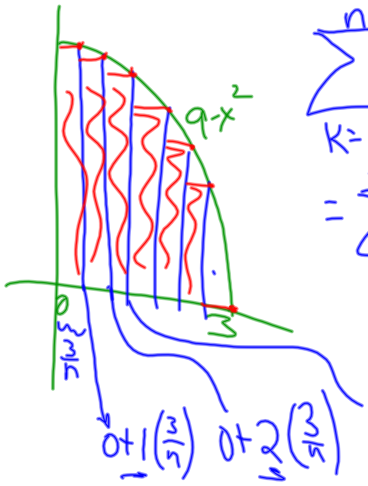
6.4/1-42



height of each rectangle WILL be the value of the function SOMEWHERE in that sub-interval!

example: Consider $y=9-x^2$



$$\sum_{k=1}^{n}\left(\frac{3}{n}\right)f\left(0+k\left(\frac{3}{n}\right)\right)$$

$$=\sum_{k=1}^{n}\left(\frac{3}{n}\right)\left(9-\left(\frac{3k}{n}\right)^2\right)$$

$$=\sum_{k=1}^{n}\frac{3}{n}\left(9-\frac{9k^2}{n^2}\right)=\sum_{k=1}^{n}\left(\frac{27}{n}-\frac{27k^2}{n^3}\right)$$

$$\boxed{\sum_{k=1}^{n}\frac{27}{n}=n\left(\frac{27}{n}\right)=27 \qquad \underbrace{\frac{27}{n}+\frac{27}{n}+\frac{27}{n}+\cdots+\frac{27}{n}}_{n}}$$

$$=\sum_{k=1}^{n}\frac{27}{n}-\sum_{k=1}^{n}\frac{27}{n^3}(k^2)$$

$$=27-\sum_{k=1}^{n}\frac{27}{n^3}(k^2)$$

$$\sum_{k=1}^{n}\frac{27}{n^3}(k^2)$$

$$\frac{27}{n^3}(1^2)+\frac{27}{n^3}(2^2)+\cdots+\frac{27}{n^3}(n^2)$$

$$=27-\frac{27}{n^3}\sum_{k=1}^{n}k^2$$

p. 3×132
$3^2\cdot 44$
$2^2\cdot 3^2\cdot 11$
396

$$=27-\frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$=27-\frac{9n(n+1)(2n+1)}{2n^3}$$

$$=27-\frac{9(n+1)(2n+1)}{2n^2}$$

Area of n rectangles superimposed on the region UNDER $y=9-x^2$ from 0 to 3

$$\lim_{n\to\infty} 27-\frac{9(n+1)(2n+1)}{2n^2}=27-\lim_{n\to\infty}\frac{9\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{2}$$

$$=27-\frac{9\cdot 2}{2}=27-9=18$$

Dec 14-7:35 AM

Preliminary definition

The area under the curve $y = f(x)$ [[and above the x-axis, also assuming $f(x) \geq 0$]] between $x = a$ and $x = b$ is

$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{b-a}{n} f(x_k^*)$$

where x_k^* is an x-value along the base of the k^{th} rectangle.

Dec 14-7:35 AM

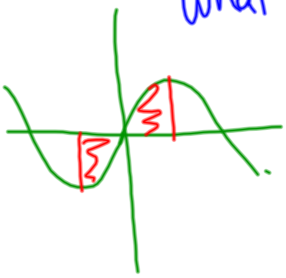
New definition AND new idea

Net Signed Area

is $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\underbrace{\frac{b-a}{n}}_{\text{"}\Delta x\text{"}}f(x_k^*)$

[[? No restriction on $f(x)$ being non-negative]]

What is the net signed area between x-axis and $y = \sin(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$?



new H/W: 6.4 / 1-41 ODD

$\sum_{K=1}^{n} K^2$

	n=1	n=2	n=3	n=4	n=5	
	1	5	14	30	55	...
diff 1		4	9	16	25	...
diff 2			5	7	9	11 ...
diff 3				2	2	2 ...

there will be an n^3 term in formula

I know the coefficient will be $\frac{2}{3 \cdot 2 \cdot 1}$

difference

new HW: 6.4 / 1–41 ODD

$$\sum_{k=1}^{n} k^2$$

	n=1	n=2	n=3	n=4	n=5	
	1	5	14	30	55	...
$-\left(\frac{2}{6}n^3:\right.$	$\frac{2}{6}$	$\frac{16}{6}$	$\frac{54}{6}$	$\frac{128}{6}$	$\frac{250}{6}$	$\left.\right)$
new seq	$\frac{4}{6}$	$\frac{14}{6}$	$\frac{30}{6}$	$\frac{52}{6}$	$\frac{80}{6}$	
diff 1		$\frac{10}{6}$	$\frac{16}{6}$	$\frac{22}{6}$	$\frac{28}{6}$	
diff 2			$\frac{6}{6}$	$\frac{6}{6}$	$\frac{6}{6}$	

there will be an n^3 term in formula

I know the coefficient will be $\frac{2}{3\cdot2\cdot1}$

difference

Coeff of n^2 $\frac{1}{2\cdot1}=\frac{1}{2}$

new HW: 6.4 / 1-41 ODD

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
new seq	$\frac{4}{6}$	$\frac{14}{6}$	$\frac{30}{6}$	$\frac{52}{6}$	$\frac{80}{6}$
$-\frac{1}{2}n^2$	$\frac{1}{2}$	2	$\frac{9}{2}$	8	$\frac{25}{2}$
newnew seq.	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$
diff		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Shazamm!

$$\sum_{k=1}^{n} k^2 \qquad \frac{2}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(2n^2+3n+1)}{6}$$

$$= \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}$$

$$f(x) = \frac{2}{6}x^3$$

$$f'(x) = \frac{2}{6}(3x^2) = \frac{6}{6}x^2 = x^2$$

$$f''(x) = 2x$$

$$f'''(x) = 2$$

Dec 14-8:42 AM